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Interface plasmon modes of local quasi-periodic superlattices

Nian-hua Liu †‡, Wei-guo Feng† and Xiang Wu†§

† Pohl Institute of Solid State Physics, Tongji University, Shanghai 200092, People's Republic of China

‡ Department of Physics, Jian Teachers Training College, Jian, Jiangxi 343009, People's Republic of China

§ Centre of Theoretical Physics, CCAST (World Laboratory), Beijing and International Centre for Materials Physics, Academia Sinica, Shengyang, People's Republic of China

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Abstract. By using the transfer-matrix method, we have studied the interface plasmon modes of an infinite-layered two-dimensional electron-gas system, in which all the layers are arranged periodically except for one region of quasi-periodic layers mapping the rule of the Fibonacci sequence. It has been found that with the removal of translational symmetry, the spectrum of localized interface plasmon modes, both acoustic-like and optic-like, becomes very rich. The dispersion relation of the interface plasmons is presented in terms of the transfer-matrix elements, and a couple of critical wavevectors is also found.

1. Introduction

The collective excitations of superlattices have been extensively studied in the past few years [1–16]. Plasmons in superlattices, as we know, can propagate along the superlattice direction because of the coupling between the layered two-dimensional electron gases (2DEGs) due to the long-range Coulomb interaction. For the periodic semiconductor superlattices (for example, the modulation-doped GaAs–AlGaAs heterostructures), the dispersion relation of bulk plasmons, in which the modes propagating along the superlattice direction are characterized by Bloch indices, has been obtained by both hydrodynamic theory [1] and the self-consistent field approach [2], and confirmed experimentally [3]. By imposing the standard electrodynamic boundary conditions at the layers of the 2DEG and taking the non-retarded limit ($qc \gg \omega$) Giuliani and Quinn [4] have investigated the surface plasmon for a semi-infinite semiconductor superlattice adjoining a bulk insulator. Their theoretical results show that the plasmon mode can occur either above or below the bulk plasmon continuum, but it exists only when the background dielectric constant of the semiconductor differs from that of the bulk material. They also showed that it intersects the bulk plasmon continuum when the wavevector is smaller than some critical value. Both the electric response of a semi-infinite layered 2DEG and Raman scattering from its bulk and surface plasmon have been worked out by solving the random-approximation equations [7]. Constantinou and Cottam [6] have recently extended the theoretical work, and calculated the collective plasmon modes of a superlattice composed of 2DEG layers separated alternately by two kinds of medium,

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which may have different thicknesses and dielectric constants. The surface plasmon modes have been investigated for various surface parameters: the surface electron density and the dielectric constant of the bulk medium. They have also examined the effects on the surface plasmon intensity due to the presence of a capping layer at the surface of a superlattice, which relates to the problem of observing surface plasmons by techniques such as Raman scattering.

Recently, Bloss has studied the localized interface plasmon modes of an infinite periodic array of quantum wells where all the wells are doped with the same electron density except for one in the middle of the system [14], and two coupled semi-infinite periodic arrays of quantum wells separated by a distance d [15]. A unique set of interface modes resulting from the break of the translational symmetry have been found.

In this paper, we extend Bloss' work by replacing the region separating the two semi-infinite superlattices with a quasi-periodic superlattice of quantum wells to study the localized interface plasmons. We let the quasi-periodic array be arranged in agreement with the Fibonacci sequence, since this heterostructure has in recent years attracted considerable interest for its scaling behaviour and critical properties [10, 11, 17–19]. We found that with increasing generation number, the interface plasmon modes (both acoustic-like and optic-like) gain a rich structure, and we also found a couple of critical wavevectors for the existence of the interface plasmon modes. It is known that the bulk plasmons of superlattices have already been observed experimentally [3,5], but those at the surfaces and interfaces have not been successfully detected yet. Several authors have remarked on the observability by techniques such as Raman scattering and electron-energy-loss spectroscopy [4–6], and using far infrared attenuated total reflection spectroscopy [16]. We hope that experimentalists are encouraged to investigate our theoretical results.

2. Theory

The system under consideration is composed of two semi-infinite superlattices of quantum wells growing in the z-direction and separated by an nth generation Fibonacci superlattice of quantum wells, in which the adjacent wells are separated by two distances a_L and a_S recursively following the Fibonacci sequence, as shown in figure 1 (for details of the Fibonacci superlattice model, we refer the reader to the pioneering work of Merlin and co-workers [19]). For simplicity, we take the two semi-infinite superlattice to have the same periodicity d and assume that the separation between the adjacent wells is so large that the overlap of the electron wavefunctions can be neglected, and that the width of the wells is so small that the electron gas there can be regarded as composed of sheets. So the system is specialized as a series of layered 2DEGs aligned parallel to the x–y plane and imbedded in a material of background dielectric constant ϵ [2, 4]. We also take the non-retarded limit and long-wavelength approximation.

To find the interface plasmon excitations, one should solve the Poisson equation coupled to the density response of the 2DEG. The general solution for the system can be written as

$$\phi_l^+(x) = \{A_l^+ \exp[-q(z - z_l)] + B_l^+ \exp[q(z - z_l)]\} \times \exp[\mathrm{i}(q \cdot r - \omega t)] \qquad z_l < z < z_{l+1} \tag{1}$$

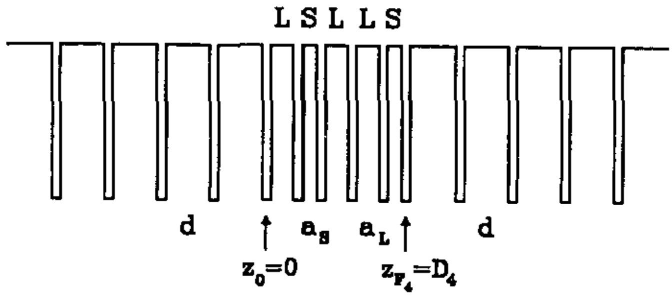

Figure 1. Two superlattices of quantum wells with the same periodicity d separated by a fourth-generation Fibonacci superlattice of quantum wells.

for $z > 0$, and

$$\phi_l^-(x) = \{A_l^- \exp[q(z - z_l)] + B_l^- \exp[-q(z - z_l)]\} \times \exp[i(q \cdot r - \omega t)] \qquad z_{l+1} < z < z_l \tag{2}$$

for $z < 0$, where r and q are the position vector and wavevector in the x–y plane respectively. The combination of the electromagnetic boundary conditions across the electron layers leads to the transfer relation [10]:

$$\begin{bmatrix} A_{l+1}^{\pm} \\ B_{l+1}^{\pm} \end{bmatrix} = \mathsf{T}(d_l) \begin{bmatrix} A_l^{\pm} \\ B_l^{\pm} \end{bmatrix} \tag{3}$$

where

$$\mathsf{T}(d_l) = \begin{bmatrix} (1+\chi)e^{-qd_l} & \chi e^{qd_l} \\ -\chi e^{-qd_l} & (1-\chi)e^{qd_l} \end{bmatrix} \tag{4}$$

with

$$\det[\mathsf{T}(d_l)] = 1 \qquad d_l = d,\ a_{\mathrm{L}},\ a_{\mathrm{S}} \tag{5}$$

and χ is the susceptibility of the 2DEG, which can be expressed as

$$\chi = \omega_p^2(q)/\omega^2 \tag{6}$$

in the long-wavelength limit. Here

$$\omega_p(q) = (2\pi e^2 nq/\epsilon m^*)^{1/2} \tag{7}$$

is the 2D plasma frequency, and m^* and n are the electron effective mass and density of the 2DEG, respectively. Across the interface at $z = 0$, the transfer relation is given by

$$\begin{bmatrix} A_0^+ \\ B_0^+ \end{bmatrix} = \mathsf{Z} \begin{bmatrix} A_0^- \\ B_0^- \end{bmatrix} \tag{8}$$

where

$$\mathbf{Z} = \begin{bmatrix} \chi & 1+\chi \\ 1-\chi & -\chi \end{bmatrix}. \tag{9}$$

For the region of the two semi-infinite periodic superlattices of quantum wells, the complex Bloch index $\kappa = k + \mathrm{i}\alpha$ can be introduced, which leads to the relation

$$\begin{bmatrix} A_{l+1}^{\pm} \\ B_{l+1}^{\pm} \end{bmatrix} = \mathrm{e}^{\mathrm{i}\kappa d} \begin{bmatrix} A_{l}^{\pm} \\ B_{l}^{\pm} \end{bmatrix}. \tag{10}$$

The real part k of κ represents the bulk mode propagating along the z-direction and the imaginary part α is a decay factor corresponding to the local mode. From equations (3)–(5) and (10), we have

$$\cos(\kappa d) = \eta \qquad \eta \equiv \tfrac{1}{2}\,\mathrm{Tr}\,\mathbf{T}(d). \tag{11}$$

It should be noted that the right-hand side of equation (11) is a real number. If the parameter $|\eta| \leqslant 1$, then $\alpha = 0$, and equation (11) gives the dispersion relation for the bulk plasmon

$$\omega(q,k) = \omega_p(q)\,[\sinh(qd)/[\cosh(qd) - \cos(kd)]]^{1/2}. \tag{12}$$

If $|\eta| > 1$, then $\alpha \neq 0$, thus kd must be equal to 0 or π depending on whether $\eta > 1$, or $\eta < -1$, and this case occurs in the forbidden band of bulk plasmon. Since we are only interested in the local interface plasmon modes, we take the case of $\alpha \neq 0$. For given values of q and ω, the decay factor α, which is the inverse of the localization length, must be positive and satisfy the equation (11). Hence we have

$$\alpha = d^{-1}\ln\left[|\eta| + (\eta^2 - 1)^{1/2}\right] \tag{13}$$

where $|\eta| > 1$.

When connecting (A_0^-, B_0^-) to $(A_{F_n}^+, B_{F_n}^+)$ for two neighbouring layers of the nth Fibonacci superlattice, we find

$$\begin{bmatrix} A_{F_n}^{+} \\ B_{F_n}^{+} \end{bmatrix} = \mathbf{X}_n \begin{bmatrix} A_{0}^{-} \\ B_{0}^{-} \end{bmatrix} \tag{14}$$

where F_n is the nth order of the Fibonacci number, which is defined as

$$F_0 = 1, \qquad F_1 = 1, \qquad F_2 = 2, \ldots, \qquad F_n = F_{n-1} + F_{n-2}; \tag{15}$$

and

$$\mathbf{X}_n = \mathbf{C}_n \mathbf{Z} \tag{16}$$

with

$$\mathbf{C}_0 = \mathbf{T}(a_S) \qquad \mathbf{C}_1 = \mathbf{T}(a_L)\ldots \qquad \mathbf{C}_n = \mathbf{C}_{n-2}\mathbf{C}_{n-1}. \tag{17}$$

After solving equations (3)–(14), we obtain the dispersion relation for interface plasmon modes as follows:

$$X_{12}\gamma^2 + (X_{11} - X_{22})\gamma - X_{21} = 0 \tag{18}$$

where X_{ij} are the elements of the transfer matrix $\mathbf{X}_n$, and

$$\gamma_{\pm} = \frac{\pm\mathrm{e}^{-\alpha d} - T_{11}(d)}{T_{12}(d)} \tag{19}$$

where $T_{ij}(d)$ is also the matrix element of $\mathbf{T}(d)$. The sign of $\gamma_{\pm}$ is determined by whether $\eta > 1$ or $\eta < -1$, corresponding to the higher-frequency modes (optic-like) and lower-frequency modes (acoustic-like), respectively.

3. Numerical results and conclusions

By using equation (18) obtained in the last section, one can calculate the dispersion relation of the interface plasmon numerically. In our real calculation, we choose the frequency unit as

$$\omega_0 = (4\pi e^2 n/\epsilon m^* d)^{1/2} \tag{20}$$

and take the ratio of the width for the two elementary cells of the Fibonacci superlattice as the inverse of the golden mean, $a_S/a_L = (\sqrt{5}-1)/2 \equiv \sigma$, and $a_L/d \equiv \tau$, where d is the periodicity of the semi-infinite superlattices.

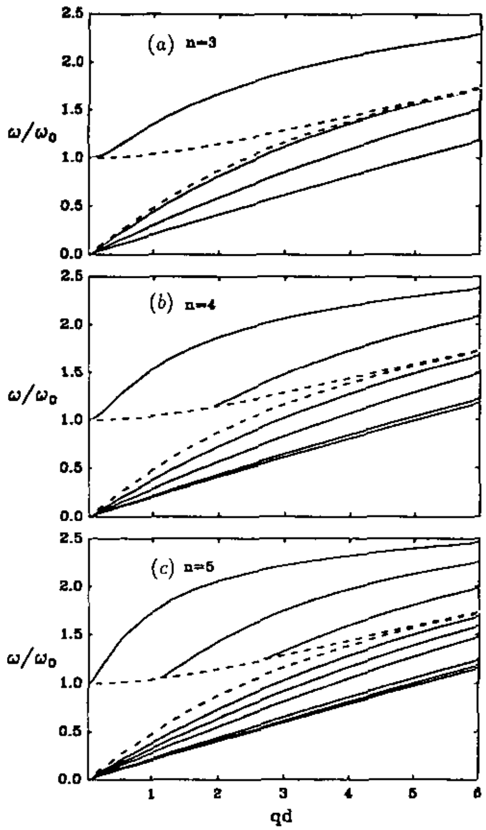

Figure 2. Plots of frequency versus qd for the three-generation local quasi-periodic superlattices: (*a*) $n = 3$, (*b*) $n = 4$ and (*c*) $n = 5$ with $a_L/d \equiv \tau = 0.2$.

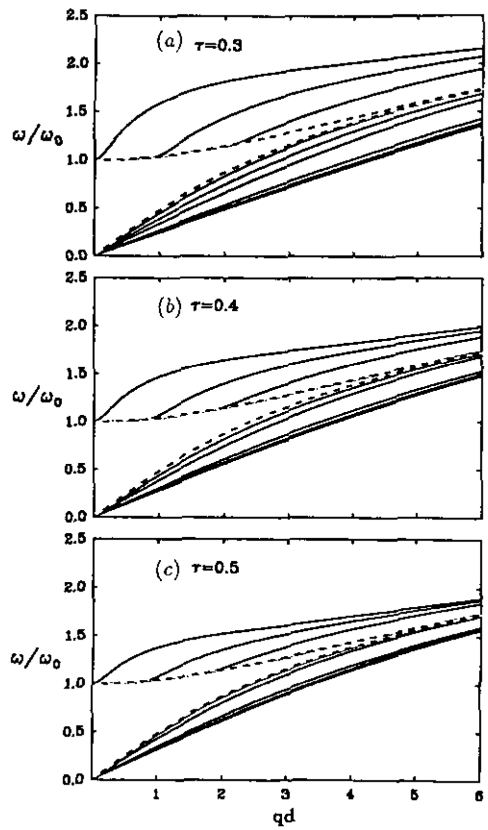

Figure 3. Plots of frequency versus qd for the fifth-generation local quasi-periodic superlattice with the different separation ratios $\tau \equiv a_L/d$: (*a*) $\tau = 0.3$, (*b*) $\tau = 0.4$ and (*c*) $\tau = 0.5$.

On the three frames of figure 2, we have plotted the plasmon dispersion for the case of $\tau = 0.2$, for generation number $n = 3$, 4 and 5 respectively. In figure 2, the region between the dashed lines, which are drawn using the dispersion equation (12) when $\cos(kd) = \pm 1$, represents the bulk plasmon continuum. We find that as generation number increases, more and more interface plasmon modes, of which there are $F_n + 1$ in all, appear in both the higher-frequency and lower-frequency region. And we also find that when we increase the value of τ, i.e. increase the separation between the 2DEGs in the Fibonacci superlattice, which means decreasing the coupling between them, all the interface modes are close to the bulk plasmon continuum. If we put the generation number $n = 2$ and $\tau = 1$, the model system will reduce to that of two semi-infinite periodic superlattices separated by a distance a_S, and the results return to those obtained by Bloss [15].

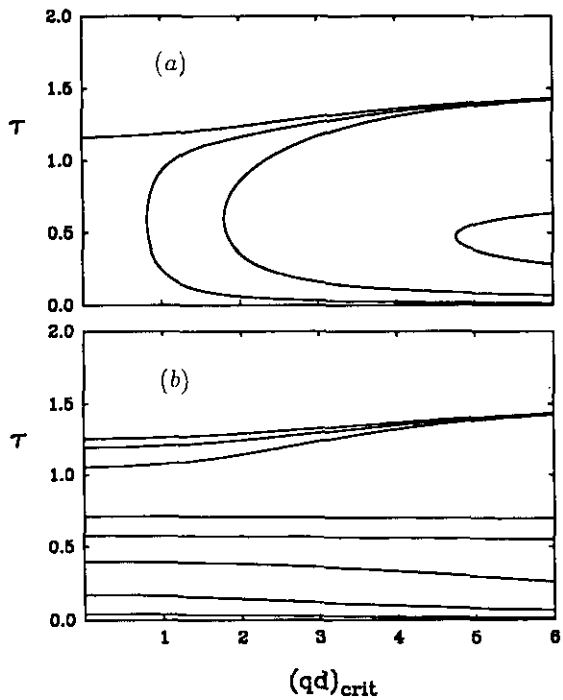

Figure 4. Plots of separation ratio $a_L/d \equiv \tau$ versus $(qd)_{\rm crit}$ determining the critical wavevector for the existence of the interface plasmon modes: (*a*) higher-frequency mode, (*b*) lower-frequency mode.

From our numerical results, we also find a couple of critical wavevectors for the existence of the interface plasmon modes as shown in figures 2 and 3. There are two kinds of interface plasmon modes as can be seen from the figures, one exists for all wavevectors and the other exists only for the case when the wavevector is greater than the critical wavevector. Letting the decay factor α approach zero in equation (18), we can study numerically the dependence of critical values of $(qd)_{\rm crit}$ on τ. In figure 4, we have plotted the curves of τ versus $(qd)_{\rm crit}$ for the case of $n = 5$. Figure 4(*a*) is relevant to the higher-frequency modes, and figure 4(*b*) to the lower-frequency modes. From figure 4, we can clearly understand the shift of the critical points (the points of dispersion curves of localized plasmon modes in contact with the edge of the bulk plasmon continuum) with τ. It can be seen that for the higher-frequency modes, all except one of the minima of the critical wavevectors are located at $\tau \simeq 0.5$.

With decreasing τ, the critical wavevector increases quickly to infinity, which means the corresponding branch of localized plasmon modes vanishes. But the topmost of the higher-frequency branches, which originates from $q = 0$, always exists as τ tends to zero. This limit case corresponds to the case that in the middle of the layered 2DEG system, there is an electron layer, whose charge density is different from the others[14]. On the other hand, if τ increases, the critical wavevectors will also become large quickly, and when τ approaches a critical value, all the critical wavevectors will increase to infinity, that is, all the localized plasmon modes, both higher-frequency and lower-frequency modes, will disappear. It is shown that whether the localized plasmon modes exist or not depends sensitively upon the geometry of the quasi-periodic superlattice.

In conclusion, we have studied the dispersion relation of the localized interface plasmon in a local quasi-periodic superlattice. The presence of these modes depends on the geometry of the quasi-periodic superlattice. If the separation a_L (or a_S) between 2DEGs in the quasi-periodic region is smaller than the periodicity of the superlattice, the rich localized interface plasmon modes appear around the two sides of the bulk plasmon continuum. When the separation a_L (or a_S) is larger than a critical value, all the interface plasmon modes disappear.

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